

4D Camera Object Tracking for Faster Vehicles (*preliminary*)

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<http://www.infinitedelta.com/wp/camera-object-tracking.pdf>
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1 Introduction

This paper addresses requirements, algorithms, and test cases for high speed vehicles and their navigation and control. Object Closure Time (OCT) algorithm extends the range of LIDAR and some common vision algorithms at a lower cost and with ranges beyond 200 meters. Depending on the application, many sensors can be eliminated.

1.1 Identification

\$HeadURL: svn+ssh://infinitedelta.com/svn/papers/camera-object-tracking/camera-object-tracking.tex
\$Id: camera-object-tracking.tex 120 2016-12-16 19:25:01Z ty \$

1.2 Applications

Applications can cover a wide range from basic driver assistance through full unmanned vehicles and fleet operations:

- Motorcycles
- Railroad
- Avionics: Sense and Avoid (a major aviation requirement!)
- UAV, UGV, Drones, etc
- Autonomous Driving
- Driver assistance
- Factory automation
- Racing

1.3 Considerations

Many real world issues including visibility, vibration and alignment can be computationally corrected. Why “4D”? Vehicle dynamics modeling has long shown knowing an object’s 3D position isn’t sufficient for dynamic situations. A full 4D representation that includes how the object position changes over time is required. The PBS Nova DARPA grand challenge special (B) and proposals (C) highlights many of these early issues.

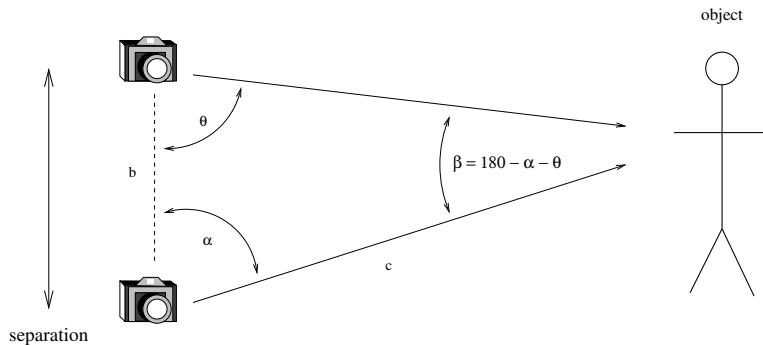
Considerations to include:

- Fast DSP image processing techniques and some issues.
- Object tracking, ranging, and closure rates.
- Navigation
- Collision avoidance.
- Networked fleet operations.
- Costs and reliability.
- Compare via graph binocular vs closure time
- Pot hole test cases.

1.4 Overview

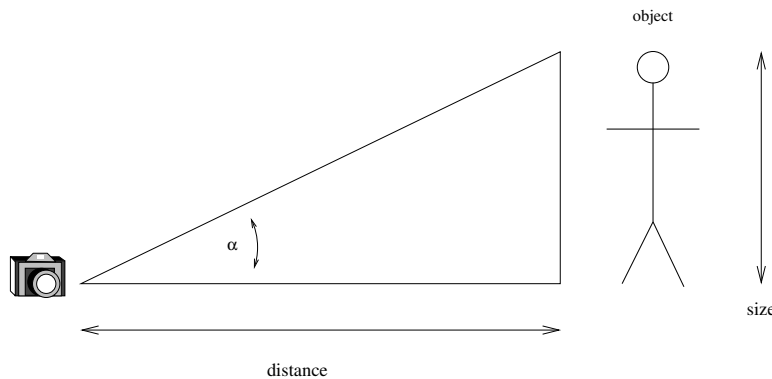
Typically there are several simultaneous triangulation solutions being processed for camera navigation and object detection. These are divided into two top level groups: The first triangulation uses

images from multiple camera positions. The “binocular” or “stereoscopic” design uses two cameras which calculates an instantaneous and is accurate for short ranges:



A moving vehicle can use a time/distance sequence of images to perform a similar triangulation, with a different separation and angle parameters.

The second triangulation group is based on the target object’s size. Two object size methods are used, the first is by knowing the object’s physical size, which requires an recognition application.



Knowing the object size can give an instantaneous range. An object’s closure rate does not require recognizing the object, only following the object from a few sequences of images.

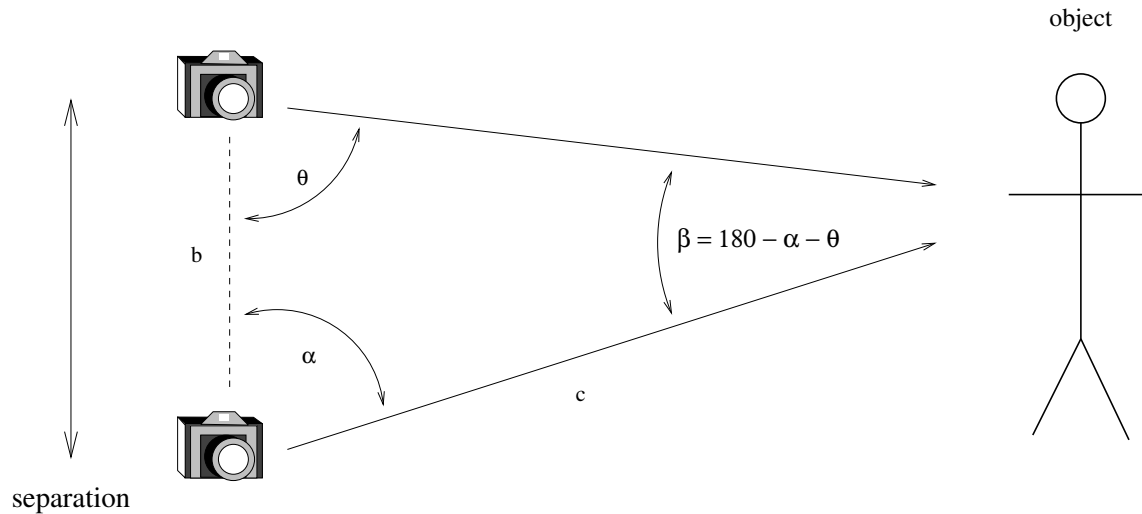
Pre-processed images are critical to all the triangulation solutions. Pre-processing shall include adjacent pixel differentiation with tracking tags.

1.4.1 Object Closure Time (OCT)

Multiple sequential images from a single camera can determine the closure time to an unknown object (OCT). The rate at which an object’s apparent size increases determines the OCT. OCT can supplement “binocular” designs or used independently. OCT extends the range over “binocular” or “stereoscopic” designs, which is required for the faster moving vehicles. OCT calculations are also faster, more robust, and easier than object recognition designs, which find objects in a database for its actual physical size. Finally, OCT design meets the timing requirements for faster vehicle’s navigation and control.

2 Multiple image positions

Multiple camera images from different positions allows a distance to an identifiable object corner. This is regardless of using multiple cameras or multiple images on a moving camera, or any combination thereof.



The 2D Law of Sines (A) determines the distance:

$$\frac{b}{\sin(\beta)} = \frac{c}{\sin(\Theta)} \tag{1}$$

$$\frac{b}{\sin(180 - \alpha - \Theta)} \times \sin(\Theta) = c = \text{distance} \tag{2}$$

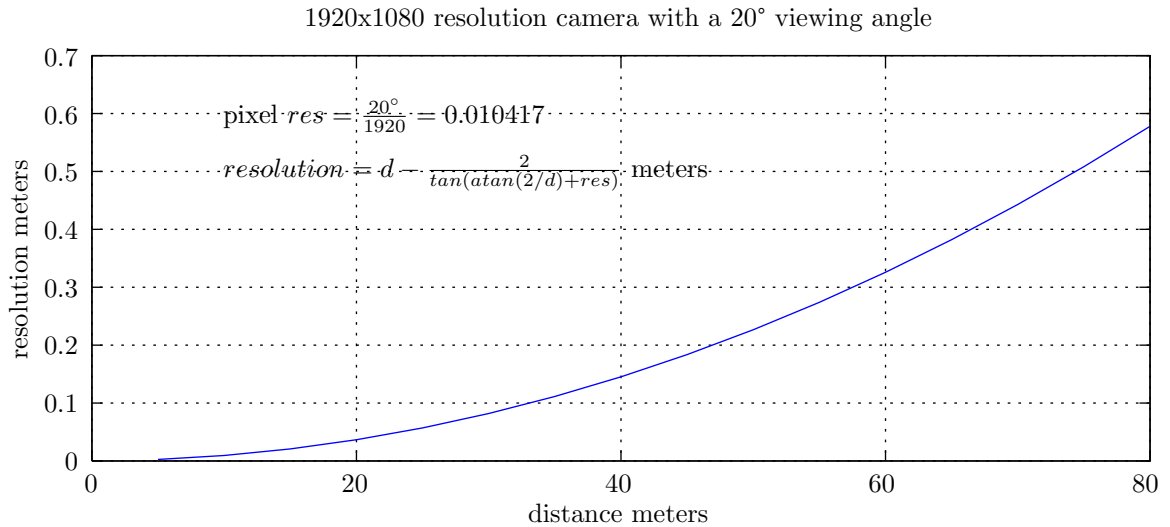
2.1 Multiple cameras

Using multiple cameras allows nearly instantaneous ranging of objects including static non-moving objects. Any pairing of two physical cameras (*binocular*) allows static ranging of track-able objects (4.3).

The object distance accuracy increases as the distance to the object drops in a multiple physical camera solution.

2.1.1 Two camera binocular example

Two 1920 pixel resolution cameras with 20 degree viewing angles separated by 2 meters has a ± 0.15 meter accuracy at nearly 40 meters:



Note that a cross camera calibration is required for non-perpendicular edges.

2.2 Multiple images on a moving camera

A moving camera allows the same binocular range calculations. It allows tracking of slower moving objects or distant objects with better precision. However, it can get thrown off by close or fast moving objects.

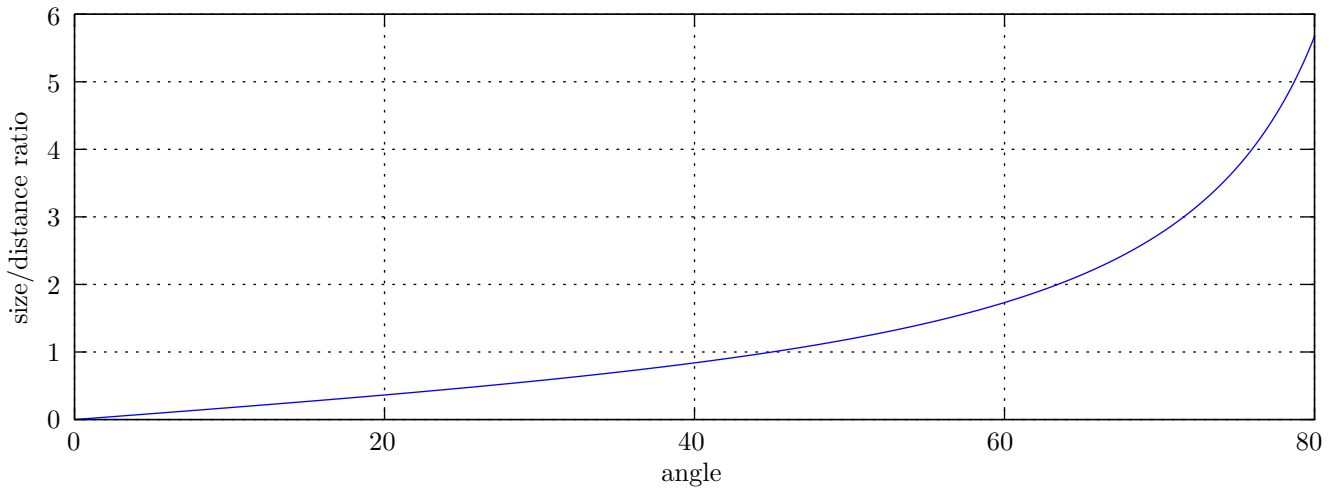
3 Object size

The tangent function definition represents the size to distance relationship:

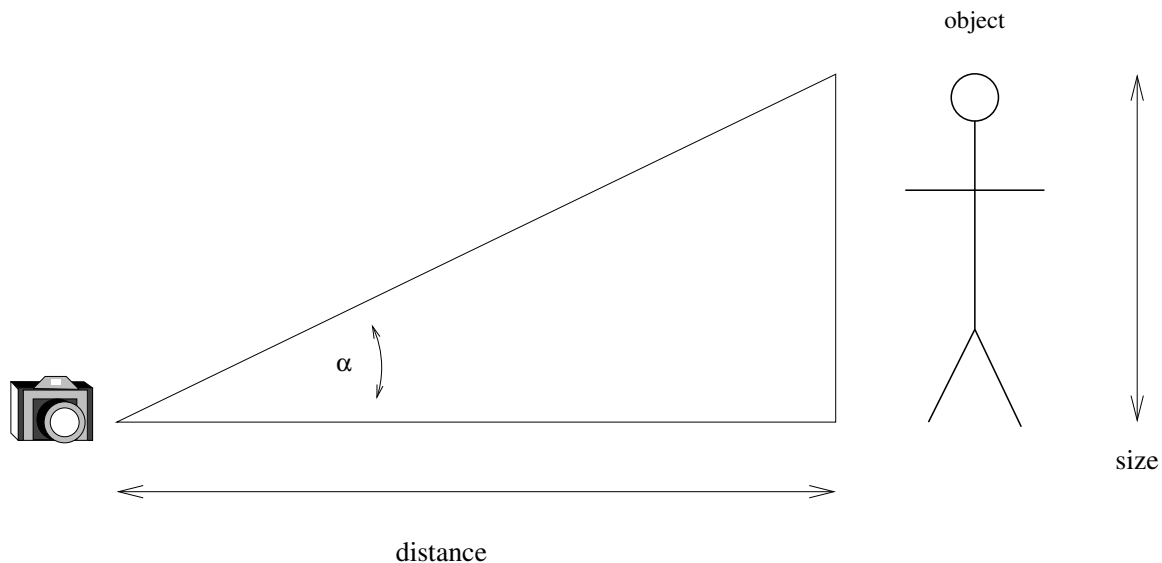
$$\tan(\alpha) = \frac{\textit{opposite}}{\textit{adjacent}} \tag{3}$$

This relationship is core to determining all distances based on size:

$$r \equiv \tan(\alpha)$$



An object's relative distance can be determined from its apparent size. This method has been used by navigators for hundreds of years. Our own perception of an object's distance is primarily based on its size, especially as the distances grow large.



The low mounting of the camera isn't required, but could be typical of a ground vehicle's design,

and is used in the following sections.

3.1 Absolute distance

An absolute distance can be easily calculated if the object's actual physical size is known. The object's image size, via calibrated camera 4.1, is converted into a usable angle “ α ” for calculations:

$$\tan(\alpha) = \frac{\textit{size}}{\textit{distance}} \quad (4)$$

3.2 Object Closure Time (OCT)

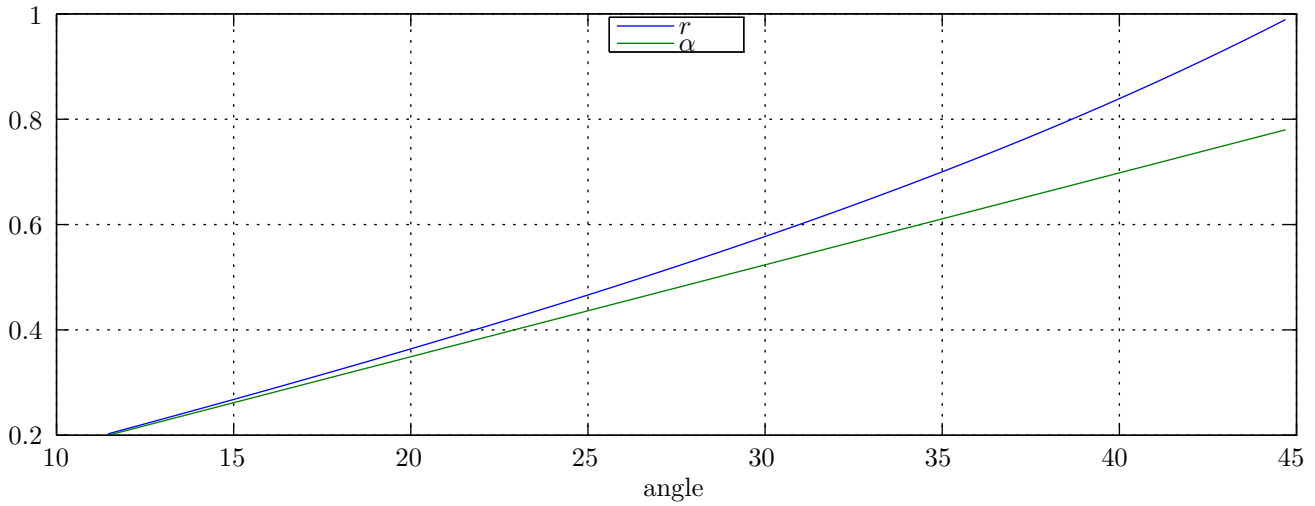
Object Closure Time (OCT), also referred to as a approach rate, can be estimated without knowing an object's actual physical size. The key is how quickly the image grows on the above tangent curve 3: Accurately calculating the closure time depends on accurately tracking and monitoring the object's image size (α). It can be sufficiently accurate for effective navigation and object avoidance. Most importantly, it doesn't require slow and expensive computationally expensive object recognition algorithms. This section makes the following assumptions:

- Sufficient contrast to generate blobs and any associations.
- Smaller objects not seen early, poses a proportionally smaller risk.
- The acceleration change between image samples isn't significant.
- Only one dimension is required for this section.
- The change in perspective also isn't significant.

Again the approach rate defines ratio “ r ”:

$$r \equiv \tan(\alpha) = \frac{\textit{size}}{\textit{distance}} \rightarrow \textit{size/distance ratio} \quad (5)$$

Where “ r ” is the size/distance **ratio** on the tangent curve 3 for determining object closure rate. The ratio “ r ” is nearly linear at small angles:



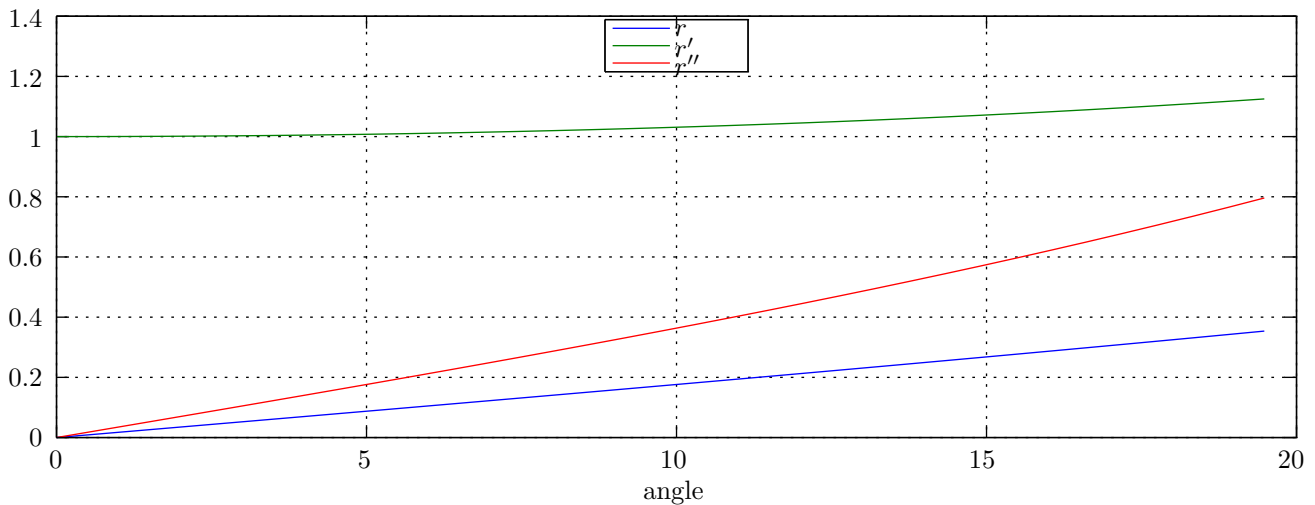
By definition: a linear function has only one slope. Hence, the first differential (slope) of the tangent function looks very linear especially at small α values:

$$r' = 1 + \tan^2(\alpha) \tag{6}$$

However, the second differential “ r'' ” (how fast the image is changing, aka acceleration) is far more significant:

$$r'' = 2 \times \tan(\alpha) \times (1 + \tan^2(\alpha)) \tag{7}$$

The following graph compares r , r' , and r'' using a camera with a 100mm lens ($\simeq 35mm$ camera D) which has a 20° viewing angle:



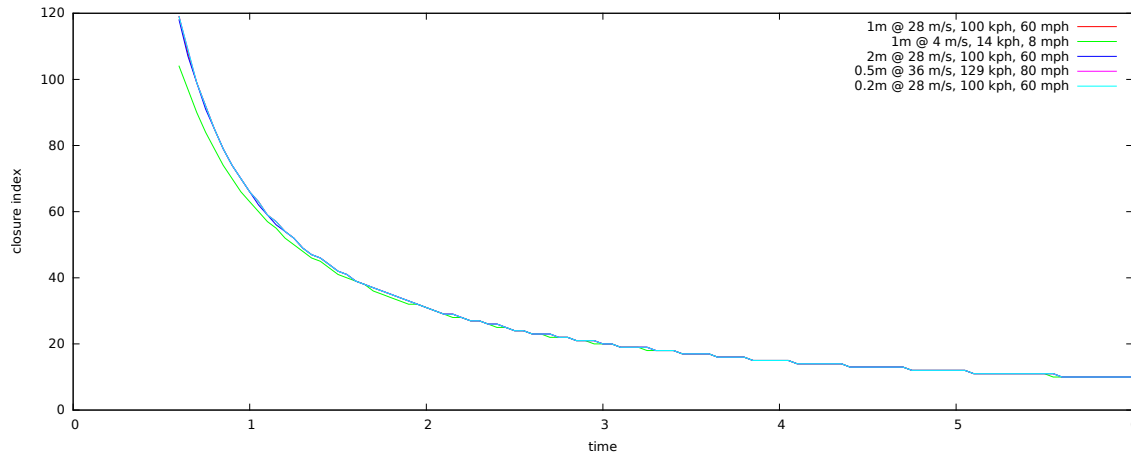
Using r'' (acceleration) effectively doubles accuracy for following the tangent curve 3.

3.2.1 Changes in r'' determines closure time

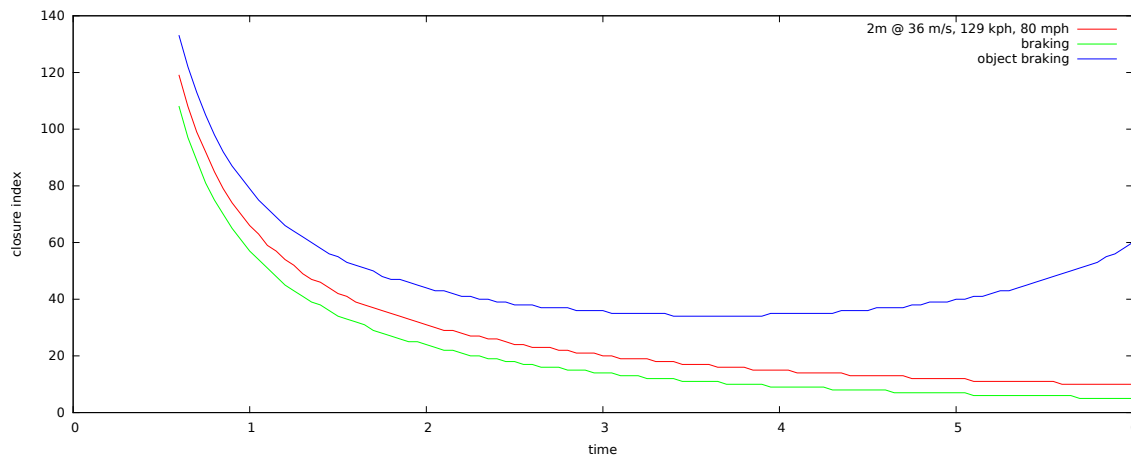
Changes in r'' directly correlates to the closure time and be directly scaled for software:

$$closure\ index \equiv 600 \times \frac{r'' - r''_{n-1}}{r''} \tag{8}$$

The following graph shows the closure time against the closure index for several different object sizes and approach speeds:



The following graph shows the closure time against the closure index under heavy braking:



This graph demonstrates relative consistency even while braking. Its especially important to note when the object is braking, an even more conservative index is reported. Now confident in a few test cases (6), a more complete verification is required.

3.2.2 Verification

The above object size theory requires an extended set of test cases to cover a wide range of the object sizes, speeds, all at different closure rates. Each of the following subsections cover a specific closure time. **Notes:**

- The blanks in the tables where r'' didn't vary enough, or when the object was either too close, or too far away for determining the object size.
- Each of these tables has an horizontal line representing minimally required point for full braking.

Closing time in 0.2 seconds: The following table shows the relative change in r'' for an object at 0.2 seconds away. The columns contains the closing velocity, distance in meters, target physical size in meters, and under heavy acceleration ($0.1m^{-1}$).

m/s	kph	mph	Dist	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	2.5	+g	-g
0.2	0	0	0.04	92	42	27	20	16	13	11	9	8	7	7	4	3	-113	23
0.4	1	0	0.08	197	92	58	42	33	27	23	20	17	16	14	9	7	-173	42
0.6	2	1	0.12	289	144	92	66	52	42	36	31	27	24	22	14	11	-236	70
0.8	2	1	0.16	362	197	127	92	71	58	49	42	37	33	30	20	15	-297	103
1	3	2	0.2	416	245	162	118	92	75	63	54	47	42	38	25	19	0	133
2	7	4	0.4	535	416	316	245	197	162	137	118	103	92	82	54	40	0	242
3	10	6	0.6	568	496	416	346	289	245	211	184	162	144	130	85	63	1623	311
4	14	8	0.8	581	535	476	416	362	316	277	245	219	197	178	118	87	1258	358
5	18	11	1	588	556	512	463	416	372	333	300	270	245	224	151	112	1065	391
6	21	13	1.2	591	568	535	496	455	416	379	346	316	289	266	184	137	957	417
7	25	15	1.4	593	576	550	519	484	450	416	384	355	328	304	215	162	888	437
8	28	17	1.6	595	581	561	535	506	476	445	416	388	362	338	245	187	841	453
9	32	20	1.8	596	585	568	547	522	496	469	442	416	391	368	273	211	808	466
10	36	22	2	597	588	574	556	535	512	488	463	439	416	393	300	234	782	477
12	43	26	2.4	597	591	581	568	553	535	516	496	476	455	435	346	277	746	495
14	50	31	2.8	598	593	586	576	564	550	535	519	502	484	467	384	316	722	508
16	57	35	3.2	598	595	589	581	572	561	549	535	521	506	491	416	350	704	518
18	64	40	3.6	599	596	591	585	577	568	558	547	535	522	509	442	379	691	526
20	72	44	4	599	597	593	588	581	574	566	556	546	535	524	463	405	681	533
22	79	49	4.4	599	597	594	590	585	578	571	563	554	545	535	481	426	673	538
24	86	53	4.8	599	597	595	591	587	581	575	568	561	553	544	496	445	666	543
26	93	58	5.2	599	598	596	593	589	584	579	573	566	559	551	508	462	661	547
28	100	62	5.6	599	598	596	593	590	586	581	576	570	564	557	519	476	656	551
30	108	67	6	599	598	597	594	591	588	584	579	574	568	562	528	488	652	554
32	115	71	6.4	599	598	597	595	592	589	586	581	577	572	567	535	499	648	556
34	122	76	6.8	599	598	597	595	593	590	587	583	579	575	570	542	508	645	559
36	129	80	7.2	599	599	597	596	594	591	588	585	581	577	573	547	516	643	561
38	136	85	7.6	599	599	598	596	594	592	590	587	583	580	576	552	523	640	563
40	144	89	8	599	599	598	597	595	593	590	588	585	581	578	556	529	638	565
45	162	100	9	599	599	598	597	596	594	592	590	588	585	582	564	542	634	568
50	180	111	10	599	599	598	598	597	595	594	592	590	588	585	571	552	630	571
55	198	123	11	599	599	599	598	597	596	595	593	592	590	588	575	560	627	574
60	216	134	12	599	599	599	598	597	597	595	594	593	591	590	579	566	625	576
65	234	145	13	599	599	599	598	598	597	596	595	594	593	591	582	570	623	578
70	252	156	14	599	599	599	599	598	597	597	596	595	593	592	584	574	621	579
75	270	167	15	599	599	599	599	598	598	597	596	595	594	593	586	577	620	580
80	288	178	16	599	599	599	599	598	598	597	597	596	595	594	588	580	618	582

1: The accelerations values are based on constant acceleration therefore are at different distances.

Closing time in 0.3 seconds: The following table shows the relative change in r'' for an object at 0.3 seconds away. The columns contains the closing velocity, distance in meters, target physical size in meters, and under heavy acceleration ($0.1m^{-1}$).

m/s	kph	mph	Dist	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	2.5	+g	-g
0.2	0	0	0.06	93	44	28	20	16	13	11	9	8	7	7	4	3	-60	11
0.4	1	0	0.12	173	93	60	44	34	28	23	20	18	16	14	9	7	-95	21
0.6	2	1	0.18	222	137	93	68	53	44	37	32	28	25	22	15	11	-137	35
0.8	2	1	0.24	248	173	123	93	73	60	51	44	38	34	31	20	15	-188	52
1	3	2	0.3	264	201	150	116	93	76	65	56	49	44	39	26	19	-249	67
2	7	4	0.6	290	264	232	201	173	150	131	116	103	93	84	56	41	0	120
3	10	6	0.9	295	282	264	243	222	201	182	165	150	137	126	87	65	1188	154
4	14	8	1.2	297	290	278	264	248	232	216	201	187	173	161	116	88	730	177
5	18	11	1.5	298	293	285	276	264	252	239	226	213	201	189	142	110	578	194
6	21	13	1.8	298	295	290	282	274	264	254	243	232	222	211	165	131	505	206
7	25	15	2.1	299	296	292	287	280	272	264	255	246	237	228	185	150	462	216
8	28	17	2.4	299	297	294	290	284	278	271	264	256	248	240	201	167	434	224
9	32	20	2.7	299	297	295	292	287	282	277	271	264	257	250	214	182	414	231
10	36	22	3	299	298	296	293	290	285	281	276	270	264	258	226	195	399	237
12	43	26	3.6	299	298	297	295	292	290	286	282	278	274	269	243	216	379	245
14	50	31	4.2	299	299	298	296	294	292	290	287	283	280	276	255	232	365	252
16	57	35	4.8	299	299	298	297	296	294	292	290	287	284	281	264	244	356	257
18	64	40	5.4	299	299	298	297	296	295	293	292	290	287	285	271	254	349	261
20	72	44	6	299	299	299	298	297	296	295	293	291	290	288	276	261	343	265
22	79	49	6.6	299	299	299	298	297	296	295	294	293	291	290	279	267	339	268
24	86	53	7.2	299	299	299	298	298	297	296	295	294	292	291	282	271	335	270
26	93	58	7.8	299	299	299	299	298	297	297	296	295	293	292	285	275	332	272
28	100	62	8.4	299	299	299	299	298	298	297	296	295	294	293	287	278	329	274
30	108	67	9	299	299	299	299	298	298	297	297	296	295	294	288	281	327	276
32	115	71	9.6	299	299	299	299	298	298	298	297	296	296	295	290	283	325	277
34	122	76	10.2	299	299	299	299	299	298	298	297	297	296	295	291	285	324	278
36	129	80	10.8	299	299	299	299	299	298	298	297	297	296	296	292	286	322	279
38	136	85	11.4	299	299	299	299	299	298	298	298	297	297	296	292	287	321	280
40	144	89	12	299	299	299	299	299	299	298	298	297	297	296	293	289	320	281
45	162	100	13.5	299	299	299	299	299	299	298	298	298	297	297	294	291	318	283
50	180	111	15	299	299	299	299	299	299	299	298	298	298	298	295	292	316	285
55	198	123	16.5	299	299	299	299	299	299	299	299	298	298	298	296	294	314	286
60	216	134	18	299	299	299	299	299	299	299	299	299	298	298	297	295	313	287
65	234	145	19.5	299	299	299	299	299	299	299	299	299	299	298	297	295	312	288
70	252	156	21	299	299	299	299	299	299	299	299	299	299	298	297	296	311	289
75	270	167	22.5	299	299	299	299	299	299	299	299	299	299	299	298	296	310	290
80	288	178	24	299	299	299	299	299	299	299	299	299	299	299	298	297	309	290

1: The accelerations values are based on constant acceleration therefore are at different distances.

Closing time in 0.4 seconds: The following table shows the relative change in r'' for an object at 0.4 seconds away. The columns contains the closing velocity, distance in meters, target physical size in meters, and under heavy acceleration ($0.1m^{-1}$).

m/s	kph	mph	Dist	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	2.5	+g	-g
0.2	0	0	0.08	89	44	28	21	16	13	11	10	8	8	7	4	3	-35	6
0.4	1	0	0.16	146	89	60	44	35	28	24	21	18	16	15	10	7	-55	12
0.6	2	1	0.24	170	123	89	68	54	44	38	32	28	25	23	15	11	-79	21
0.8	2	1	0.32	182	146	113	89	72	60	51	44	39	35	31	21	15	-107	31
1	3	2	0.4	188	160	132	108	89	75	65	56	50	44	40	26	20	-142	40
2	7	4	0.8	196	188	175	160	146	132	119	108	98	89	82	56	42	0	74
3	10	6	1.2	198	194	188	179	170	160	151	141	132	123	115	84	65	1510	96
4	14	8	1.6	199	196	193	188	182	175	168	160	153	146	139	108	86	630	112
5	18	11	2	199	197	195	192	188	183	178	172	166	160	155	127	104	450	123
6	21	13	2.4	199	198	196	194	191	188	184	179	175	170	165	141	119	375	132
7	25	15	2.8	199	198	197	195	193	191	188	184	181	177	173	152	132	335	139
8	28	17	3.2	199	199	198	196	195	193	190	188	185	182	178	160	142	310	144
9	32	20	3.6	199	199	198	197	196	194	192	190	188	185	182	167	151	292	149
10	36	22	4	199	199	198	197	196	195	193	192	190	188	185	172	157	280	153
12	43	26	4.8	199	199	199	198	197	196	195	194	193	191	189	179	168	263	159
14	50	31	5.6	199	199	199	198	198	197	196	195	194	193	192	184	175	251	164
16	57	35	6.4	199	199	199	199	198	198	197	196	196	195	194	188	180	244	168
18	64	40	7.2	199	199	199	199	199	198	198	197	196	196	195	190	184	238	171
20	72	44	8	199	199	199	199	199	198	198	197	197	196	196	192	186	233	174
22	79	49	8.8	199	199	199	199	199	199	198	198	197	197	196	193	189	230	176
24	86	53	9.6	199	199	199	199	199	199	198	198	198	197	197	194	190	227	177
26	93	58	10.4	199	199	199	199	199	199	199	198	198	198	197	195	191	225	179
28	100	62	11.2	199	199	199	199	199	199	199	198	198	198	198	195	193	223	180
30	108	67	12	199	199	199	199	199	199	199	199	198	198	198	196	193	221	182
32	115	71	12.8	199	199	199	199	199	199	199	199	198	198	198	196	194	220	183
34	122	76	13.6	199	199	199	199	199	199	199	199	199	198	198	197	195	218	183
36	129	80	14.4	199	199	199	199	199	199	199	199	199	199	198	197	195	217	184
38	136	85	15.2	199	199	199	199	199	199	199	199	199	199	198	197	196	216	185
40	144	89	16	199	199	199	199	199	199	199	199	199	199	199	197	196	215	186
45	162	100	18	199	199	199	199	199	199	199	199	199	199	199	198	197	213	187
50	180	111	20	199	199	199	199	199	199	199	199	199	199	199	198	197	212	188
55	198	123	22	199	199	199	199	199	199	199	199	199	199	199	198	198	211	189
60	216	134	24	199	199	199	199	199	199	199	199	199	199	199	199	198	210	190
65	234	145	26	199	199	199	199	199	199	199	199	199	199	199	199	198	209	191
70	252	156	28	199	199	199	199	199	199	199	199	199	199	199	199	198	208	191
75	270	167	30	199	199	199	199	199	199	199	199	199	199	199	199	198	208	192
80	288	178	32	199	199	199	199	199	199	199	199	199	199	199	199	199	207	192

1: The accelerations values are based on constant acceleration therefore are at different distances.

Closing time in 0.5 seconds: The following table shows the relative change in r'' for an object at 0.5 seconds away. The columns contains the closing velocity, distance in meters, target physical size in meters, and under heavy acceleration ($0.1m^{-1}$).

m/s	kph	mph	Dist	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	2.5	+g	-g
0.2	0	0	0.1	84	45	29	21	16	13	11	10	9	8	7	4	3	-22	4
0.4	1	0	0.2	122	84	59	45	35	29	24	21	18	16	15	10	7	-35	8
0.6	2	1	0.3	136	108	84	66	54	45	38	33	29	26	23	15	11	-49	14
0.8	2	1	0.4	141	122	102	84	70	59	51	45	39	35	32	21	16	-66	21
1	3	2	0.5	144	131	114	98	84	73	63	56	50	45	40	27	20	-87	27
2	7	4	1	148	144	138	131	122	114	106	98	91	84	78	56	42	-285	51
3	10	6	1.5	149	147	144	140	136	131	125	120	114	108	103	80	63	0	67
4	14	8	2	149	148	146	144	141	138	134	131	127	122	118	98	81	699	79
5	18	11	2.5	149	149	148	146	144	142	139	137	134	131	127	111	95	415	87
6	21	13	3	149	149	148	147	146	144	142	140	138	136	133	120	106	324	94
7	25	15	3.5	149	149	148	148	147	146	144	143	141	139	137	126	114	279	100
8	28	17	4	149	149	149	148	147	146	145	144	143	141	140	131	120	253	104
9	32	20	4.5	149	149	149	148	148	147	146	145	144	143	142	134	125	235	108
10	36	22	5	149	149	149	149	148	148	147	146	145	144	143	137	129	223	111
12	43	26	6	149	149	149	149	149	148	148	147	146	146	145	140	134	206	116
14	50	31	7	149	149	149	149	149	148	148	148	147	147	146	143	138	196	120
16	57	35	8	149	149	149	149	149	149	148	148	148	147	147	144	141	189	123
18	64	40	9	149	149	149	149	149	149	149	148	148	148	147	145	142	183	126
20	72	44	10	149	149	149	149	149	149	149	149	148	148	148	146	144	179	128
22	79	49	11	149	149	149	149	149	149	149	149	149	149	148	148	147	176	129
24	86	53	12	149	149	149	149	149	149	149	149	149	149	148	147	145	174	131
26	93	58	13	149	149	149	149	149	149	149	149	149	149	149	147	146	172	132
28	100	62	14	149	149	149	149	149	149	149	149	149	149	149	149	148	170	133
30	108	67	15	149	149	149	149	149	149	149	149	149	149	149	149	148	168	134
32	115	71	16	149	149	149	149	149	149	149	149	149	149	149	149	148	167	135
34	122	76	17	149	149	149	149	149	149	149	149	149	149	149	149	148	166	136
36	129	80	18	149	149	149	149	149	149	149	149	149	149	149	149	148	165	137
38	136	85	19	149	149	149	149	149	149	149	149	149	149	149	149	149	164	137
40	144	89	20	149	149	149	149	149	149	149	149	149	149	149	149	149	163	138
45	162	100	22.5	149	149	149	149	149	149	149	149	149	149	149	149	149	162	139
50	180	111	25	149	149	149	149	149	149	149	149	149	149	149	149	149	160	140
55	198	123	27.5	149	149	149	149	149	149	149	149	149	149	149	149	149	159	141
60	216	134	30	149	149	149	149	149	149	149	149	149	149	149	149	149	158	141
65	234	145	32.5	149	149	149	149	149	149	149	149	149	149	149	149	149	158	142
70	252	156	35	149	149	149	149	149	149	149	149	149	149	149	149	149	157	143
75	270	167	37.5	149	149	149	149	149	149	149	149	149	149	149	149	149	157	143
80	288	178	40	149	149	149	149	149	149	149	149	149	149	149	149	149	156	143

1: The accelerations values are based on constant acceleration therefore are at different distances.

Closing time in 1 seconds: The following table shows the relative change in r'' for an object at 1 seconds away. The columns contains the closing velocity, distance in meters, target physical size in meters, and under heavy acceleration ($0.2m^{-1}$).

m/s	kph	mph	Dist	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	2.5	+g	-g
0.2	0	0	0.2	56	40	29	22	17	14	12	10	9	8	7	5	3	-5	1
0.4	1	0	0.4	63	56	47	40	33	29	25	22	19	17	15	10	8	-8	2
0.6	2	1	0.6	65	61	56	50	45	40	35	32	29	26	24	16	12	-11	3
0.8	2	1	0.8	65	63	60	56	51	47	43	40	36	33	31	22	16	-14	5
1	3	2	1	66	64	62	59	56	52	49	46	43	40	37	27	21	-18	7
2	7	4	2	66	66	65	64	63	62	60	59	57	56	54	46	38	-42	15
3	10	6	3	66	66	66	65	65	64	63	63	62	61	60	55	49	-87	21
4	14	8	4	66	66	66	66	65	65	65	64	64	63	63	59	55	-188	26
5	18	11	5	66	66	66	66	66	65	65	65	65	64	64	61	58	0	30
6	21	13	6	66	66	66	66	66	66	65	65	65	65	64	63	60	728	33
7	25	15	7	66	66	66	66	66	66	66	65	65	65	65	64	62	312	36
8	28	17	8	66	66	66	66	66	66	66	66	66	65	65	64	63	216	38
9	32	20	9	66	66	66	66	66	66	66	66	66	66	65	65	63	174	40
10	36	22	10	66	66	66	66	66	66	66	66	66	66	66	65	64	150	42
12	43	26	12	66	66	66	66	66	66	66	66	66	66	66	65	65	124	44
14	50	31	14	66	66	66	66	66	66	66	66	66	66	66	65	65	111	47
16	57	35	16	66	66	66	66	66	66	66	66	66	66	66	66	65	102	48
18	64	40	18	66	66	66	66	66	66	66	66	66	66	66	66	65	97	50
20	72	44	20	66	66	66	66	66	66	66	66	66	66	66	66	66	92	51
22	79	49	22	66	66	66	66	66	66	66	66	66	66	66	66	66	89	52
24	86	53	24	66	66	66	66	66	66	66	66	66	66	66	66	66	87	53
26	93	58	26	66	66	66	66	66	66	66	66	66	66	66	66	66	85	54
28	100	62	28	66	66	66	66	66	66	66	66	66	66	66	66	66	83	55
30	108	67	30	66	66	66	66	66	66	66	66	66	66	66	66	66	82	55
32	115	71	32	66	66	66	66	66	66	66	66	66	66	66	66	66	81	56
34	122	76	34	66	66	66	66	66	66	66	66	66	66	66	66	66	80	57
36	129	80	36	66	66	66	66	66	66	66	66	66	66	66	66	66	79	57
38	136	85	38	66	66	66	66	66	66	66	66	66	66	66	66	66	78	57
40	144	89	40	66	66	66	66	66	66	66	66	66	66	66	66	66	77	58
45	162	100	45	66	66	66	66	66	66	66	66	66	66	66	66	66	76	59
50	180	111	50	66	66	66	66	66	66	66	66	66	66	66	66	66	75	59
55	198	123	55	66	66	66	66	66	66	66	66	66	66	66	66	66	74	60
60	216	134	60	66	66	66	66	66	66	66	66	66	66	66	66	66	73	60
65	234	145	65	66	66	66	66	66	66	66	66	66	66	66	66	66	73	61
70	252	156	70	66	66	66	66	66	66	66	66	66	66	66	66	66	72	61
75	270	167	75	66	66	66	66	66	66	66	66	66	66	66	66	66	72	61
80	288	178	80	66	66	66	66	66	66	66	66	66	66	66	66	66	71	62

1: The accelerations values are based on constant acceleration therefore are at different distances.

Closing time in 1.5 seconds: The following table shows the relative change in r'' for an object at 1.5 seconds away. The columns contains the closing velocity, distance in meters, target physical size in meters, and under heavy acceleration ($0.2m^{-1}$).

m/s	kph	mph	Dist	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	2.5	+g	-g
0.2	0	0	0.3	39	32	26	21	17	14	12	10	9	8	7	5	3	-2	0
0.4	1	0	0.6	41	39	36	32	29	26	23	21	19	17	15	10	8	-3	1
0.6	2	1	0.9	42	41	39	37	35	32	30	28	26	24	22	16	12	-4	1
0.8	2	1	1.2	42	41	40	39	38	36	34	32	31	29	27	21	16	-6	2
1	3	2	1.5	42	42	41	40	39	38	37	35	34	32	31	25	20	-7	3
2	7	4	3	42	42	42	42	41	41	41	40	40	39	39	35	32	-16	7
3	10	6	4.5	42	42	42	42	42	42	42	41	41	41	41	39	37	-28	10
4	14	8	6	42	42	42	42	42	42	42	42	42	41	41	40	39	-46	13
5	18	11	7.5	42	42	42	42	42	42	42	42	42	42	42	41	40	-77	15
6	21	13	9	42	42	42	42	42	42	42	42	42	42	42	41	41	-140	17
7	25	15	10.5	42	42	42	42	42	42	42	42	42	42	42	42	41	-338	19
8	28	17	12	42	42	42	42	42	42	42	42	42	42	42	42	41	2145	20
9	32	20	13.5	42	42	42	42	42	42	42	42	42	42	42	42	42	354	22
10	36	22	15	42	42	42	42	42	42	42	42	42	42	42	42	42	207	23
12	43	26	18	42	42	42	42	42	42	42	42	42	42	42	42	42	127	25
14	50	31	21	42	42	42	42	42	42	42	42	42	42	42	42	42	99	26
16	57	35	24	42	42	42	42	42	42	42	42	42	42	42	42	42	85	28
18	64	40	27	42	42	42	42	42	42	42	42	42	42	42	42	42	77	29
20	72	44	30	42	42	42	42	42	42	42	42	42	42	42	42	42	71	30
22	79	49	33	42	42	42	42	42	42	42	42	42	42	42	42	42	67	31
24	86	53	36	42	42	42	42	42	42	42	42	42	42	42	42	42	64	31
26	93	58	39	42	42	42	42	42	42	42	42	42	42	42	42	42	62	32
28	100	62	42	42	42	42	42	42	42	42	42	42	42	42	42	42	60	33
30	108	67	45	42	42	42	42	42	42	42	42	42	42	42	42	42	58	33
32	115	71	48	42	42	42	42	42	42	42	42	42	42	42	42	42	57	34
34	122	76	51	42	42	42	42	42	42	42	42	42	42	42	42	42	56	34
36	129	80	54	42	42	42	42	42	42	42	42	42	42	42	42	42	55	34
38	136	85	57	42	42	42	42	42	42	42	42	42	42	42	42	42	54	35
40	144	89	60	42	42	42	42	42	42	42	42	42	42	42	42	42	53	35
45	162	100	67.5	42	42	42	42	42	42	42	42	42	42	42	42	42	52	36
50	180	111	75	42	42	42	42	42	42	42	42	42	42	42	42	42	51	36
55	198	123	82.5	42	42	42	42	42	42	42	42	42	42	42	42	42	50	37
60	216	134	90	42	42	42	42	42	42	42	42	42	42	42	42	42	49	37
65	234	145	97.5	42	42	42	42	42	42	42	42	42	42	42	42	42	48	38
70	252	156	105	42	42	42	42	42	42	42	42	42	42	42	42	42	48	38
75	270	167	112.5	42	42	42	42	42	42	42	42	42	42	42	42	42	48	38
80	288	178	120	42	42	42	42	42	42	42	42	42	42	42	42	42	47	38

1: The accelerations values are based on constant acceleration therefore are at different distances.

Closing time in 2 seconds: The following table shows the relative change in r'' for an object at 2 seconds away. The columns contains the closing velocity, distance in meters, target physical size in meters, and under heavy acceleration ($1m^{-1}$).

m/s	kph	mph	Dist	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	2.5	+g	-g
0.2	0	0	0.4	30	26	23	19	16	14	12	10	9	8	7	5	3	-1	0
0.4	1	0	0.8	31	30	28	26	25	23	21	19	18	16	15	10	8	-2	0
0.6	2	1	1.2	31	30	30	29	28	26	25	24	23	21	20	15	12	-2	1
0.8	2	1	1.6	31	31	30	30	29	28	27	26	26	25	24	19	16	-3	1
1	3	2	2	31	31	31	30	30	29	29	28	27	26	26	22	18	-4	2
2	7	4	4	31	31	31	31	31	31	30	30	30	30	29	28	26	-8	4
3	10	6	6	31	31	31	31	31	31	31	31	31	31	30	30	29	-13	6
4	14	8	8	31	31	31	31	31	31	31	31	31	31	31	30	30	-21	8
5	18	11	10	31	31	31	31	31	31	31	31	31	31	31	31	30	-30	9
6	21	13	12	31	31	31	31	31	31	31	31	31	31	31	31	30	-45	11
7	25	15	14	31	31	31	31	31	31	31	31	31	31	31	31	31	-67	12
8	28	17	16	31	31	31	31	31	31	31	31	31	31	31	31	31	-107	13
9	32	20	18	31	31	31	31	31	31	31	31	31	31	31	31	31	-194	14
10	36	22	20	31	31	31	31	31	31	31	31	31	31	31	31	31	0	15
12	43	26	24	31	31	31	31	31	31	31	31	31	31	31	31	31	219	16
14	50	31	28	31	31	31	31	31	31	31	31	31	31	31	31	31	121	17
16	57	35	32	31	31	31	31	31	31	31	31	31	31	31	31	31	89	18
18	64	40	36	31	31	31	31	31	31	31	31	31	31	31	31	31	74	19
20	72	44	40	31	31	31	31	31	31	31	31	31	31	31	31	31	65	20
22	79	49	44	31	31	31	31	31	31	31	31	31	31	31	31	31	60	21
24	86	53	48	31	31	31	31	31	31	31	31	31	31	31	31	31	55	21
26	93	58	52	31	31	31	31	31	31	31	31	31	31	31	31	31	52	22
28	100	62	56	31	31	31	31	31	31	31	31	31	31	31	31	31	50	22
30	108	67	60	31	31	31	31	31	31	31	31	31	31	31	31	31	48	23
32	115	71	64	31	31	31	31	31	31	31	31	31	31	31	31	31	46	23
34	122	76	68	31	31	31	31	31	31	31	31	31	31	31	31	31	45	24
36	129	80	72	31	31	31	31	31	31	31	31	31	31	31	31	31	44	24
38	136	85	76	31	31	31	31	31	31	31	31	31	31	31	31	31	43	24
40	144	89	80	31	31	31	31	31	31	31	31	31	31	31	31	31	42	24
45	162	100	90	31	31	31	31	31	31	31	31	31	31	31	31	31	41	25
50	180	111	100	31	31	31	31	31	31	31	31	31	31	31	31	31	39	26
55	198	123	110	31	31	31	31	31	31	31	31	31	31	31	31	31	39	26
60	216	134	120	31	31	31	31	31	31	31	31	31	31	31	31	31	38	26
65	234	145	130	31	31	31	31	31	31	31	31	31	31	31	31	31	37	27
70	252	156	140	31	31	31	31	31	31	31	31	31	31	31	31	31	37	27
75	270	167	150	31	31	31	31	31	31	31	31	31	31	31	31	31	36	27
80	288	178	160	31	31	31	31	31	31	31	31	31	31	31	31	31	36	27

1: The accelerations values are based on constant acceleration therefore are at different distances.

Closing time in 2.5 seconds: The following table shows the relative change in r'' for an object at 2.5 seconds away. The columns contains the closing velocity, distance in meters, target physical size in meters, and under heavy acceleration ($1m^{-1}$).

m/s	kph	mph	Dist	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	2.5	+g	-g
0.2	0	0	0.5	24	22	20	17	15	13	12	10	9	8	7	5	4	0	0
0.4	1	0	1	24	24	23	22	21	20	18	17	16	15	14	10	8	-1	0
0.6	2	1	1.5	24	24	24	23	23	22	21	20	20	19	18	14	12	-1	0
0.8	2	1	2	24	24	24	24	23	23	23	22	21	21	20	17	15	-2	1
1	3	2	2.5	24	24	24	24	24	24	23	23	22	22	22	19	17	-2	1
2	7	4	5	24	24	24	24	24	24	24	24	24	24	24	23	22	-5	2
3	10	6	7.5	24	24	24	24	24	24	24	24	24	24	24	24	23	-8	4
4	14	8	10	24	24	24	24	24	24	24	24	24	24	24	24	24	-12	5
5	18	11	12.5	24	24	24	24	24	24	24	24	24	24	24	24	24	-16	6
6	21	13	15	24	24	24	24	24	24	24	24	24	24	24	24	24	-22	7
7	25	15	17.5	24	24	24	24	24	24	24	24	24	24	24	24	24	-31	8
8	28	17	20	24	24	24	24	24	24	24	24	24	24	24	24	24	-42	9
9	32	20	22.5	24	24	24	24	24	24	24	24	24	24	24	24	24	-60	10
10	36	22	25	24	24	24	24	24	24	24	24	24	24	24	24	24	-90	10
12	43	26	30	24	24	24	24	24	24	24	24	24	24	24	24	24	-306	11
14	50	31	35	24	24	24	24	24	24	24	24	24	24	24	24	24	272	12
16	57	35	40	24	24	24	24	24	24	24	24	24	24	24	24	24	124	13
18	64	40	45	24	24	24	24	24	24	24	24	24	24	24	24	24	86	14
20	72	44	50	24	24	24	24	24	24	24	24	24	24	24	24	24	69	15
22	79	49	55	24	24	24	24	24	24	24	24	24	24	24	24	24	60	15
24	86	53	60	24	24	24	24	24	24	24	24	24	24	24	24	24	53	16
26	93	58	65	24	24	24	24	24	24	24	24	24	24	24	24	24	49	16
28	100	62	70	24	24	24	24	24	24	24	24	24	24	24	24	24	46	17
30	108	67	75	24	24	24	24	24	24	24	24	24	24	24	24	24	43	17
32	115	71	80	24	24	24	24	24	24	24	24	24	24	24	24	24	41	17
34	122	76	85	24	24	24	24	24	24	24	24	24	24	24	24	24	40	18
36	129	80	90	24	24	24	24	24	24	24	24	24	24	24	24	24	38	18
38	136	85	95	24	24	24	24	24	24	24	24	24	24	24	24	24	37	18
40	144	89	100	24	24	24	24	24	24	24	24	24	24	24	24	24	36	18
45	162	100	112.5	24	24	24	24	24	24	24	24	24	24	24	24	24	35	19
50	180	111	125	24	24	24	24	24	24	24	24	24	24	24	24	24	33	19
55	198	123	137.5	24	24	24	24	24	24	24	24	24	24	24	24	24	32	20
60	216	134	150	24	24	24	24	24	24	24	24	24	24	24	24	24	31	20
65	234	145	162.5	24	24	24	24	24	24	24	24	24	24	24	24	24	31	20
70	252	156	175	24	24	24	24	24	24	24	24	24	24	24	24	24	30	21
75	270	167	187.5	24	24	24	24	24	24	24	24	24	24	24	24	24	30	21
80	288	178	200	24	24	24	24	24	24	24	24	24	24	24	24	24	29	21

1: The accelerations values are based on constant acceleration therefore are at different distances.

Closing time in 3 seconds: The following table shows the relative change in r'' for an object at 3 seconds away. The columns contains the closing velocity, distance in meters, target physical size in meters, and under heavy acceleration ($1m^{-1}$).

m/s	kph	mph	Dist	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	2.5	+g	-g
0.2	0	0	0.6	20	19	17	16	14	12	11	10	9	8	7	5	4	0	0
0.4	1	0	1.2	20	20	19	19	18	17	16	16	15	14	13	10	8	0	0
0.6	2	1	1.8	20	20	20	20	19	19	18	18	17	17	16	13	11	-1	0
0.8	2	1	2.4	20	20	20	20	20	19	19	19	18	18	18	16	14	-1	0
1	3	2	3	20	20	20	20	20	20	19	19	19	19	18	17	15	-1	0
2	7	4	6	20	20	20	20	20	20	20	20	20	20	20	19	19	-3	2
3	10	6	9	20	20	20	20	20	20	20	20	20	20	20	20	19	-5	3
4	14	8	12	20	20	20	20	20	20	20	20	20	20	20	20	20	-7	4
5	18	11	15	20	20	20	20	20	20	20	20	20	20	20	20	20	-10	4
6	21	13	18	20	20	20	20	20	20	20	20	20	20	20	20	20	-14	5
7	25	15	21	20	20	20	20	20	20	20	20	20	20	20	20	20	-18	6
8	28	17	24	20	20	20	20	20	20	20	20	20	20	20	20	20	-23	6
9	32	20	27	20	20	20	20	20	20	20	20	20	20	20	20	20	-30	7
10	36	22	30	20	20	20	20	20	20	20	20	20	20	20	20	20	-40	8
12	43	26	36	20	20	20	20	20	20	20	20	20	20	20	20	20	-78	8
14	50	31	42	20	20	20	20	20	20	20	20	20	20	20	20	20	-226	9
16	57	35	48	20	20	20	20	20	20	20	20	20	20	20	20	20	389	10
18	64	40	54	20	20	20	20	20	20	20	20	20	20	20	20	20	134	11
20	72	44	60	20	20	20	20	20	20	20	20	20	20	20	20	20	87	11
22	79	49	66	20	20	20	20	20	20	20	20	20	20	20	20	20	67	12
24	86	53	72	20	20	20	20	20	20	20	20	20	20	20	20	20	57	12
26	93	58	78	20	20	20	20	20	20	20	20	20	20	20	20	20	50	12
28	100	62	84	20	20	20	20	20	20	20	20	20	20	20	20	20	45	13
30	108	67	90	20	20	20	20	20	20	20	20	20	20	20	20	20	42	13
32	115	71	96	20	20	20	20	20	20	20	20	20	20	20	20	20	39	13
34	122	76	102	20	20	20	20	20	20	20	20	20	20	20	20	20	37	14
36	129	80	108	20	20	20	20	20	20	20	20	20	20	20	20	20	36	14
38	136	85	114	20	20	20	20	20	20	20	20	20	20	20	20	20	34	14
40	144	89	120	20	20	20	20	20	20	20	20	20	20	20	20	20	33	14
45	162	100	135	20	20	20	20	20	20	20	20	20	20	20	20	20	31	15
50	180	111	150	20	20	20	20	20	20	20	20	20	20	20	20	20	29	15
55	198	123	165	20	20	20	20	20	20	20	20	20	20	20	20	20	28	16
60	216	134	180	20	20	20	20	20	20	20	20	20	20	20	20	20	27	16
65	234	145	195	20	20	20	20	20	20	20	20	20	20	20	20	20	27	16
70	252	156	210	20	20	20	20	20	20	20	20	20	20	20	20	20	26	16
75	270	167	225	20	20	20	20	20	20	20	20	20	20	20	20	20	26	17
80	288	178	240	20	20	20	20	20	20	20	20	20	20	20	20	20	25	17

1: The accelerations values are based on constant acceleration therefore are at different distances.

Closing time in 4 seconds: The following table shows the relative change in r'' for an object at 4 seconds away. The columns contains the closing velocity, distance in meters, target physical size in meters, and under heavy acceleration ($1m^{-1}$).

m/s	kph	mph	Dist	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	2.5	+g	-g
0.2	0	0	0.8	15	14	14	13	12	11	10	9	8	8	7	5	4	0	0
0.4	1	0	1.6	15	15	15	14	14	14	13	13	12	12	11	9	7	0	0
0.6	2	1	2.4	15	15	15	15	14	14	14	14	14	13	13	11	10	0	0
0.8	2	1	3.2	15	15	15	15	15	15	14	14	14	14	14	13	12	0	0
1	3	2	4	15	15	15	15	15	15	15	14	14	14	14	13	13	-1	0
2	7	4	8	15	15	15	15	15	15	15	15	15	15	15	14	14	-1	1
3	10	6	12	15	15	15	15	15	15	15	15	15	15	15	15	15	-2	1
4	14	8	16	15	15	15	15	15	15	15	15	15	15	15	15	15	-4	2
5	18	11	20	15	15	15	15	15	15	15	15	15	15	15	15	15	-5	2
6	21	13	24	15	15	15	15	15	15	15	15	15	15	15	15	15	-6	3
7	25	15	28	15	15	15	15	15	15	15	15	15	15	15	15	15	-8	3
8	28	17	32	15	15	15	15	15	15	15	15	15	15	15	15	15	-10	4
9	32	20	36	15	15	15	15	15	15	15	15	15	15	15	15	15	-12	4
10	36	22	40	15	15	15	15	15	15	15	15	15	15	15	15	15	-15	4
12	43	26	48	15	15	15	15	15	15	15	15	15	15	15	15	15	-23	5
14	50	31	56	15	15	15	15	15	15	15	15	15	15	15	15	15	-35	6
16	57	35	64	15	15	15	15	15	15	15	15	15	15	15	15	15	-60	6
18	64	40	72	15	15	15	15	15	15	15	15	15	15	15	15	15	-131	7
20	72	44	80	15	15	15	15	15	15	15	15	15	15	15	15	15	0	7
22	79	49	88	15	15	15	15	15	15	15	15	15	15	15	15	15	179	7
24	86	53	96	15	15	15	15	15	15	15	15	15	15	15	15	15	95	8
26	93	58	104	15	15	15	15	15	15	15	15	15	15	15	15	15	68	8
28	100	62	112	15	15	15	15	15	15	15	15	15	15	15	15	15	55	8
30	108	67	120	15	15	15	15	15	15	15	15	15	15	15	15	15	47	9
32	115	71	128	15	15	15	15	15	15	15	15	15	15	15	15	15	41	9
34	122	76	136	15	15	15	15	15	15	15	15	15	15	15	15	15	37	9
36	129	80	144	15	15	15	15	15	15	15	15	15	15	15	15	15	35	9
38	136	85	152	15	15	15	15	15	15	15	15	15	15	15	15	15	32	9
40	144	89	160	15	15	15	15	15	15	15	15	15	15	15	15	15	31	10
45	162	100	180	15	15	15	15	15	15	15	15	15	15	15	15	15	27	10
50	180	111	200	15	15	15	15	15	15	15	15	15	15	15	15	15	25	10
55	198	123	220	15	15	15	15	15	15	15	15	15	15	15	15	15	24	11
60	216	134	240	15	15	15	15	15	15	15	15	15	15	15	15	15	23	11
65	234	145	260	15	15	15	15	15	15	15	15	15	15	15	15	15	22	11
70	252	156	280	15	15	15	15	15	15	15	15	15	15	15	15	15	21	11
75	270	167	300	15	15	15	15	15	15	15	15	15	15	15	15	15	21	12
80	288	178	320	15	15	15	15	15	15	15	15	15	15	15	15	15	20	12

1: The accelerations values are based on constant acceleration therefore are at different distances.

Closing time in 5 seconds: The following table shows the relative change in r'' for an object at 5 seconds away. The columns contains the closing velocity, distance in meters, target physical size in meters, and under heavy acceleration ($1m^{-1}$).

m/s	kph	mph	Dist	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	2.5	+g	-g
0.2	0	0	1	12	11	11	11	10	9	9	8	8	7	7	5	4	0	0
0.4	1	0	2	12	12	12	11	11	11	11	11	10	10	10	8	7	0	0
0.6	2	1	3	12	12	12	12	12	11	11	11	11	11	11	10	9	0	0
0.8	2	1	4	12	12	12	12	12	12	11	11	11	11	11	11	10	0	0
1	3	2	5	12	12	12	12	12	12	12	12	11	11	11	11	10	0	0
2	7	4	10	12	12	12	12	12	12	12	12	12	12	12	12	11	-1	0
3	10	6	15	12	12	12	12	12	12	12	12	12	12	12	12	12	-1	1
4	14	8	20	12	12	12	12	12	12	12	12	12	12	12	12	12	-2	1
5	18	11	25	12	12	12	12	12	12	12	12	12	12	12	12	12	-3	1
6	21	13	30	12	12	12	12	12	12	12	12	12	12	12	12	12	-4	2
7	25	15	35	12	12	12	12	12	12	12	12	12	12	12	12	12	-4	2
8	28	17	40	12	12	12	12	12	12	12	12	12	12	12	12	12	-5	2
9	32	20	45	12	12	12	12	12	12	12	12	12	12	12	12	12	-7	3
10	36	22	50	12	12	12	12	12	12	12	12	12	12	12	12	12	-8	3
12	43	26	60	12	12	12	12	12	12	12	12	12	12	12	12	12	-11	3
14	50	31	70	12	12	12	12	12	12	12	12	12	12	12	12	12	-15	4
16	57	35	80	12	12	12	12	12	12	12	12	12	12	12	12	12	-22	4
18	64	40	90	12	12	12	12	12	12	12	12	12	12	12	12	12	-31	5
20	72	44	100	12	12	12	12	12	12	12	12	12	12	12	12	12	-49	5
22	79	49	110	12	12	12	12	12	12	12	12	12	12	12	12	12	-90	5
24	86	53	120	12	12	12	12	12	12	12	12	12	12	12	12	12	-284	5
26	93	58	130	12	12	12	12	12	12	12	12	12	12	12	12	12	315	6
28	100	62	140	12	12	12	12	12	12	12	12	12	12	12	12	12	115	6
30	108	67	150	12	12	12	12	12	12	12	12	12	12	12	12	12	74	6
32	115	71	160	12	12	12	12	12	12	12	12	12	12	12	12	12	56	6
34	122	76	170	12	12	12	12	12	12	12	12	12	12	12	12	12	46	7
36	129	80	180	12	12	12	12	12	12	12	12	12	12	12	12	12	40	7
38	136	85	190	12	12	12	12	12	12	12	12	12	12	12	12	12	36	7
40	144	89	200	12	12	12	12	12	12	12	12	12	12	12	12	12	32	7
45	162	100	225	12	12	12	12	12	12	12	12	12	12	12	12	12	27	7
50	180	111	250	12	12	12	12	12	12	12	12	12	12	12	12	12	24	8
55	198	123	275	12	12	12	12	12	12	12	12	12	12	12	12	12	22	8
60	216	134	300	12	12	12	12	12	12	12	12	12	12	12	12	12	21	8
65	234	145	325	12	12	12	12	12	12	12	12	12	12	12	12	12	19	8
70	252	156	350	12	12	12	12	12	12	12	12	12	12	12	12	12	19	8
75	270	167	375	12	12	12	12	12	12	12	12	12	12	12	12	12	18	9
80	288	178	400	12	12	12	12	12	12	12	12	12	12	12	12	12	17	9

1: The accelerations values are based on constant acceleration therefore are at different distances.

Closing time in 6 seconds: The following table shows the relative change in r'' for an object at 6 seconds away. The columns contains the closing velocity, distance in meters, target physical size in meters, and under heavy acceleration ($1m^{-1}$).

m/s	kph	mph	Dist	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	2.5	+g	-g
0.2	0	0	1.2	10	9	9	9	9	8	8	7	7	7	6	5	4	0	0
0.4	1	0	2.4	10	10	10	9	9	9	9	9	9	9	8	7	6	0	0
0.6	2	1	3.6	10	10	10	10	10	9	9	9	9	9	9	8	8	0	0
0.8	2	1	4.8	10	10	10	10	10	10	10	9	9	9	9	9	9	0	0
1	3	2	6	10	10	10	10	10	10	10	10	10	9	9	9	9	0	0
2	7	4	12	10	10	10	10	10	10	10	10	10	10	10	10	9	0	0
3	10	6	18	10	10	10	10	10	10	10	10	10	10	10	10	10	-1	0
4	14	8	24	10	10	10	10	10	10	10	10	10	10	10	10	10	-1	1
5	18	11	30	10	10	10	10	10	10	10	10	10	10	10	10	10	-2	1
6	21	13	36	10	10	10	10	10	10	10	10	10	10	10	10	10	-2	1
7	25	15	42	10	10	10	10	10	10	10	10	10	10	10	10	10	-3	1
8	28	17	48	10	10	10	10	10	10	10	10	10	10	10	10	10	-3	2
9	32	20	54	10	10	10	10	10	10	10	10	10	10	10	10	10	-4	2
10	36	22	60	10	10	10	10	10	10	10	10	10	10	10	10	10	-5	2
12	43	26	72	10	10	10	10	10	10	10	10	10	10	10	10	10	-6	2
14	50	31	84	10	10	10	10	10	10	10	10	10	10	10	10	10	-9	3
16	57	35	96	10	10	10	10	10	10	10	10	10	10	10	10	10	-11	3
18	64	40	108	10	10	10	10	10	10	10	10	10	10	10	10	10	-15	3
20	72	44	120	10	10	10	10	10	10	10	10	10	10	10	10	10	-20	4
22	79	49	132	10	10	10	10	10	10	10	10	10	10	10	10	10	-28	4
24	86	53	144	10	10	10	10	10	10	10	10	10	10	10	10	10	-41	4
26	93	58	156	10	10	10	10	10	10	10	10	10	10	10	10	10	-68	4
28	100	62	168	10	10	10	10	10	10	10	10	10	10	10	10	10	-150	4
30	108	67	180	10	10	10	10	10	10	10	10	10	10	10	10	10	1722	5
32	115	71	192	10	10	10	10	10	10	10	10	10	10	10	10	10	155	5
34	122	76	204	10	10	10	10	10	10	10	10	10	10	10	10	10	84	5
36	129	80	216	10	10	10	10	10	10	10	10	10	10	10	10	10	60	5
38	136	85	228	10	10	10	10	10	10	10	10	10	10	10	10	10	48	5
40	144	89	240	10	10	10	10	10	10	10	10	10	10	10	10	10	40	5
45	162	100	270	10	10	10	10	10	10	10	10	10	10	10	10	10	30	6
50	180	111	300	10	10	10	10	10	10	10	10	10	10	10	10	10	25	6
55	198	123	330	10	10	10	10	10	10	10	10	10	10	10	10	10	22	6
60	216	134	360	10	10	10	10	10	10	10	10	10	10	10	10	10	20	6
65	234	145	390	10	10	10	10	10	10	10	10	10	10	10	10	10	18	6
70	252	156	420	10	10	10	10	10	10	10	10	10	10	10	10	10	17	7
75	270	167	450	10	10	10	10	10	10	10	10	10	10	10	10	10	16	7
80	288	178	480	10	10	10	10	10	10	10	10	10	10	10	10	10	16	7

1: The accelerations values are based on constant acceleration therefore are at different distances.

Closing time in 7 seconds: The following table shows the relative change in r'' for an object at 7 seconds away. The columns contains the closing velocity, distance in meters, target physical size in meters, and under heavy acceleration ($1m^{-1}$).

m/s	kph	mph	Dist	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	2.5	+g	-g
0.2	0	0	1.4	8	8	8	8	8	7	7	7	6	6	6	5	4	0	0
0.4	1	0	2.8	8	8	8	8	8	8	8	8	8	8	7	7	6	0	0
0.6	2	1	4.2	8	8	8	8	8	8	8	8	8	8	8	7	7	0	0
0.8	2	1	5.6	8	8	8	8	8	8	8	8	8	8	8	8	7	0	0
1	3	2	7	8	8	8	8	8	8	8	8	8	8	8	8	8	0	0
2	7	4	14	8	8	8	8	8	8	8	8	8	8	8	8	8	0	0
3	10	6	21	8	8	8	8	8	8	8	8	8	8	8	8	8	0	0
4	14	8	28	8	8	8	8	8	8	8	8	8	8	8	8	8	-1	0
5	18	11	35	8	8	8	8	8	8	8	8	8	8	8	8	8	-1	1
6	21	13	42	8	8	8	8	8	8	8	8	8	8	8	8	8	-1	1
7	25	15	49	8	8	8	8	8	8	8	8	8	8	8	8	8	-2	1
8	28	17	56	8	8	8	8	8	8	8	8	8	8	8	8	8	-2	1
9	32	20	63	8	8	8	8	8	8	8	8	8	8	8	8	8	-3	1
10	36	22	70	8	8	8	8	8	8	8	8	8	8	8	8	8	-3	1
12	43	26	84	8	8	8	8	8	8	8	8	8	8	8	8	8	-4	2
14	50	31	98	8	8	8	8	8	8	8	8	8	8	8	8	8	-5	2
16	57	35	112	8	8	8	8	8	8	8	8	8	8	8	8	8	-7	2
18	64	40	126	8	8	8	8	8	8	8	8	8	8	8	8	8	-9	2
20	72	44	140	8	8	8	8	8	8	8	8	8	8	8	8	8	-11	3
22	79	49	154	8	8	8	8	8	8	8	8	8	8	8	8	8	-15	3
24	86	53	168	8	8	8	8	8	8	8	8	8	8	8	8	8	-19	3
26	93	58	182	8	8	8	8	8	8	8	8	8	8	8	8	8	-25	3
28	100	62	196	8	8	8	8	8	8	8	8	8	8	8	8	8	-36	3
30	108	67	210	8	8	8	8	8	8	8	8	8	8	8	8	8	-54	3
32	115	71	224	8	8	8	8	8	8	8	8	8	8	8	8	8	-99	4
34	122	76	238	8	8	8	8	8	8	8	8	8	8	8	8	8	-358	4
36	129	80	252	8	8	8	8	8	8	8	8	8	8	8	8	8	260	4
38	136	85	266	8	8	8	8	8	8	8	8	8	8	8	8	8	103	4
40	144	89	280	8	8	8	8	8	8	8	8	8	8	8	8	8	67	4
45	162	100	315	8	8	8	8	8	8	8	8	8	8	8	8	8	38	4
50	180	111	350	8	8	8	8	8	8	8	8	8	8	8	8	8	28	5
55	198	123	385	8	8	8	8	8	8	8	8	8	8	8	8	8	23	5
60	216	134	420	8	8	8	8	8	8	8	8	8	8	8	8	8	20	5
65	234	145	455	8	8	8	8	8	8	8	8	8	8	8	8	8	18	5
70	252	156	490	8	8	8	8	8	8	8	8	8	8	8	8	8	17	5
75	270	167	525	8	8	8	8	8	8	8	8	8	8	8	8	8	16	5
80	288	178	560	8	8	8	8	8	8	8	8	8	8	8	8	8	15	6

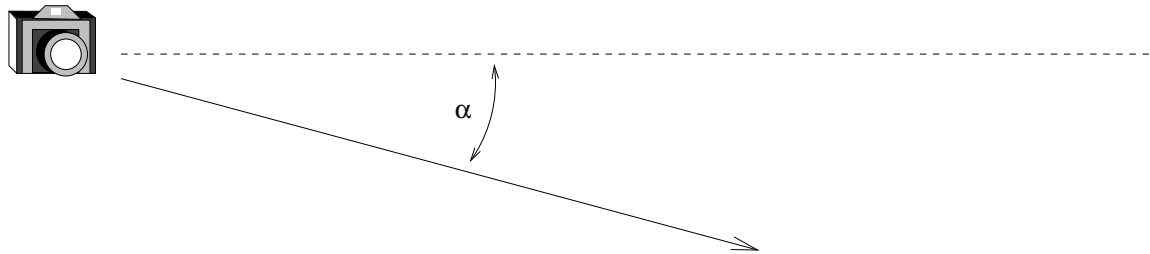
1: The accelerations values are based on constant acceleration therefore are at different distances.

4 Preprocessing

Preprocessing architecture is the key to performance and safety. Before any high level algorithm can start, each image needs to be broken into blobs including their associations with other images. Every pixel need to be calibrated and monitored for health, in the case of bugs, weather, etc. Although an initial pixel calibration is required, and continuous health monitoring with corrections are also required.

4.1 Pixel Calibration

Every pixel has a 3D vector (P_v) representing where is space relative to a camera origin:



A rough initial estimate can assume a consistent angle (P_v) variation between pixels. Many of the higher quality camera/lens configurations can nearly achieve perfect consistency. However, any zoom adjustments, or lens distortions can throw off final calculations.

4.2 Pixel Health

Weather, bugs, and many other causes can degrade the system's effectiveness. Often these are temporary, but need to be monitored closely and added/removed from the solution.

4.3 Trackable blobs

5 References

- A. 2D Law of Sines
https://en.wikipedia.org/wiki/Law_of_sines
- B. PBS Nova DARPA Grand Challenge
<http://www.pbs.org/wgbh/nova/darpa/>
- C. Image processing for DARPA's Urban Grand Challenge
http://infinitedelta.com/wp/darpa_image_thoughts.pdf
- D. Field of View (FOV) from Focal Length: $FOV = 2 \times \arctan(\frac{x}{2 \times f})$ where x is the diagonal of the film. https://en.wikipedia.org/wiki/Focal_length

6 Source

The following C++ code was used for the Object Closure Time (OCT) graph and verification (3.2.1):

```
// $Id: dist.cc 121 2016-12-16 20:25:27Z ty $
// $HeadURL: svn+ssh://infinitedelta.com/svn/papers/camera-object-tracking/dist.cc $

// Object Size Approach Time testing

#include <iostream>
#include <math.h>

// const double pixels=1080000.0;
const double pixels=1080.0;
const double maxG=9.8;
const double sampleRate=0.050; // 20 Hz
// const int maxHistory=40;
const int maxHistory=3;
const double pixelsPerRadian=pixels/20.0*180.0/M_PI;
const double siMph=1.0/0.44704;
const double siKph=1.0/0.2777777777777777;
const double closureIndexScale=600.0;
typedef double ImgTime;
int dbg=0;

struct blob // Image blob as delivered from the preprocessing
{
    struct blob *prev;
    ImgTime timeStamp;
    double vertSize;
    blob() {prev=0; timeStamp=0; vertSize = 0;}

    struct blob *setup(blob *previous, const ImgTime now, const double vsize)
    {
        prev=previous;
        timeStamp=now;
        vertSize=vsize;
        return this;
    }

    double ClosureAngle(void)
    {
        struct blob *old=this;

        while (old->prev) old = old->prev;

        if (vertSize <= old->vertSize) return 0;

        return atan(sqrt((vertSize - old->vertSize) / (timeStamp - old->timeStamp) - 1 ));
    }

    double ClosureIndex(void)
    {
        struct blob *old=this;

        while (old->prev) old = old->prev;

        if (vertSize <= old->vertSize) return 0;

        return closureIndexScale * (vertSize - old->vertSize) / vertSize;
    }
};

int test(const double vPhysicalSize, const double time, const double velocity, const double acceleration=0.0)
{
    ImgTime now=0.0;
    double dist = 0.5 * time * time * acceleration + velocity * time;
    double currentVelocity = velocity;
    struct blob imageHistory[maxHistory], *prev=0;
    int histIdx=0;

    std::cout << " & ";
    // std::cout << imageSize;

    // if (imageSize >= pixels-2 || imageSize < 10) return 0;

    // Create the previous samples based on velocity and acceleration, and at least 5 pixels in size change.

    for (histIdx=1; histIdx<maxHistory; ++histIdx)
    {
        if (currentVelocity >= 0) break;
        now -= sampleRate;
        dist -= currentVelocity * sampleRate;
        currentVelocity -= acceleration * sampleRate;
    }
}
```

```

while(--histIdx >= 0)
{
double imageSize = pixelsPerRadian * atan(vPhysicalSize/dist);
// double imageSize = (double) ((int) (1.0 * pixelsPerRadian * atan(vPhysicalSize/dist)) / 1.0);
// double imageSize = (double) ((int) (pixelsPerRadian * atan(vPhysicalSize/dist)));

prev = imageHistory[histIdx].setup(prev, now, imageSize);

if (dbg)
{
std::cout << " " << now;
std::cout << " " << dist;
std::cout << " " << currentVelocity;
std::cout << " " << imageSize;
std::cout << std::endl;
}

now += sampleRate;
currentVelocity += acceleration * sampleRate;
dist += currentVelocity * sampleRate;
}

// std::cout << imageHistory[0].ClosureTime();
std::cout << (int) -imageHistory[0].ClosureIndex();
if (dbg)
{
std::cout << std::endl;
}
return 0;
}

int main (const int argc, const char *argv[])
{
if (argc > 1 && *argv[1] == 'd')
{
++dbg;
test(1, 1, -1);
// test(1, 1, 1, -9.8);
std::cout << std::endl;
return 0;
}

if (argc > 1 && *argv[1] == 'r') // Demonstrate a few test cases for object-size-closure graph
{
for (double time=0.6; time<6.0; time += sampleRate)
{
std::cout << time;
test(1.0, time, -28);
test(1.0, time, -4);
test(2.0, time, -28);
test(0.5, time, -36);
test(0.2, time, -28);
std::cout << std::endl;
}
return 0;
}

if (argc > 1 && *argv[1] == 'a') // Demonstrate a few test cases for heavy braking
{
for (double time=0.6; time<6.0; time += sampleRate)
{
std::cout << time;
test(1.0, time, -36);
test(1.0, time, -36, -maxG);
test(1.0, time, -36, maxG);
std::cout << std::endl;
}
return 0;
}

// Attempt to complete test cases including many corner
for (double time=0.2; time<7.1; time += time < 0.5 ? 0.1 : time < 3 ? 0.5 : 1.0)
{
bool canStop=true;
double gSize = time < 1.0 ? 0.1 : time < 2.0 ? 0.2 : 1.0;

std::cout << "\\newpage" << std::endl;
std::cout << "\\paragraph{Closing time in " << time << " seconds:}" << std::endl << std::endl;
std::cout << "The following table shows the relative change in $r'$' for an object at " << time << " seconds away." << std::endl;
std::cout << "The columns contains the closing velocity, distance in meters, target physical size in meters, and under heavy acceleration (";
std::cout << gSize << "m $1$)." << std::endl << std::endl;
// std::cout << "\\vspace{1cm} " << std::endl;

// std::cout << "\\hspace{-8mm} ";
std::cout << "\\begin{center}" << std::endl;
// std::cout << "\\small" << std::endl;
std::cout << "\\footnotesize" << std::endl;
std::cout << "\\begin{tabular}{rrrr|rrrrrrrrrr|rr} \\hline" << std::endl;
std::cout <<
"m/s & kph & mph & Dist & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 1.0 & 1.5 & 2.0 & 2.5 & +g & -g \\ \\hline"

```

```

<< std::endl;

for (double velocity=0.2; velocity<82.0; velocity += velocity < 0.9 ? 0.2 : velocity < 10 ? 1.0 : velocity < 40.0 ? 2.0 : 5.0)
{
    if (canStop && velocity > maxG * time)
    {
        std::cout << "\\hline" << std::endl;
        canStop = false;
    }
    std::cout << velocity << " & " << (int) (velocity*siKph) << " & " << (int) (velocity*siMph) << " & " << (velocity*time);
    for (double size=0.1; size<2.51; size += size < 1.0 ? 0.1 : 0.5) test(size, time, -velocity);
    test(gSize, time, -velocity, maxG);
    test(gSize, time, -velocity, -maxG);
    std::cout << " \\\\" << std::endl;
}
std::cout << "\\end{tabular}" << std::endl << std::endl;
std::cout << "{\\bf 1:} The accelerations values are based on constant acceleration therefore are at different distances." << std::endl << std::endl;
std::cout << "\\end{center}" << std::endl;
}

return 0;
}

```